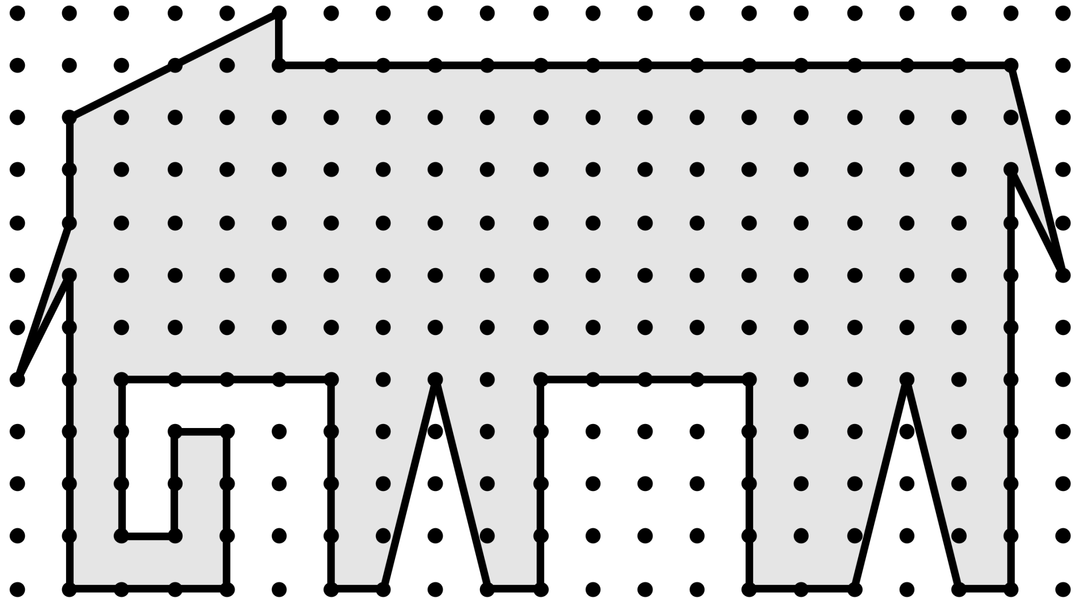
Counting Areas

Task 1:

King Bahubali loved elephants so much that he kept a herd of them. In fact, he planted his coconut garden in such a way that it looked like an elephant when viewed from his terrace! But the elephants would walk around the garden and destroy it. So the king put a fence around the garden to keep the elephants away as seen in the figure given below. The trees were planted on a square grid, with one tree at each grid point, so as to provide sufficient space for each tree. If the king's grounds were 20 units long and 11 units wide, can you find the area available for the elephants to roam, by just counting the coconut trees?

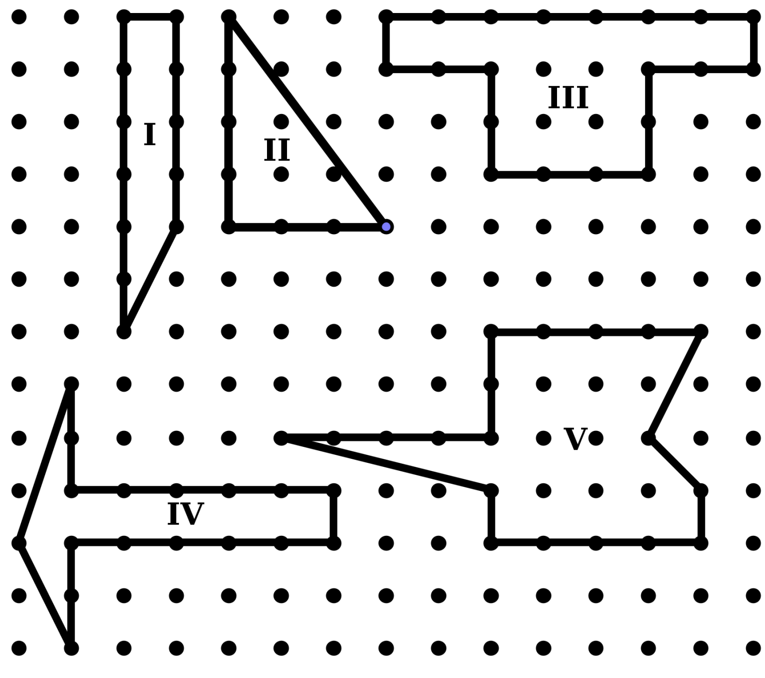
If you can't figure out, go ahead with the remaining tasks, and you will be able to do this at the end of the tasks!!



Task 2:

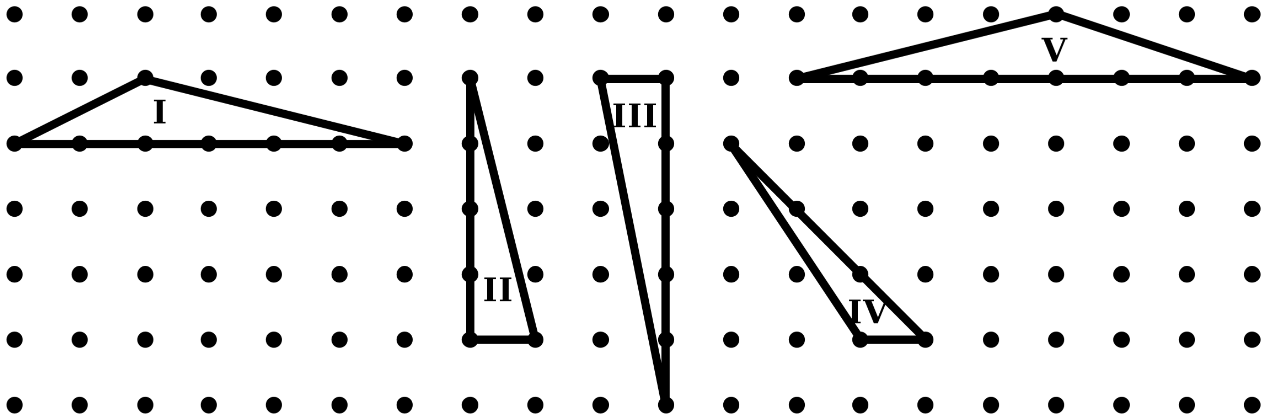
Given below are some figures. Find the area of each and complete the given table.

Figure	Area in Sq Units
I	
II	
III	
IV	
V	



Task 3

a) Find the area of the following triangles.



Also count the number of grid-points on the boundary of each triangle, and fill the table below.

Triangle	Area in Square Units	Number of grid-points on the boundary (B)
I		
II		
III		
IV		
V		

b) Do you see any relation between the area of the triangle and the number of grid-points on its boundary?

c) Does the same relation hold for figures I to V in Task 2? If not, for which ones does the relation hold?

I) The relation holds for figures ________________. (Write the number of the figure.)

ii) The relation does not hold for figures _______________. (---- ditto -----)

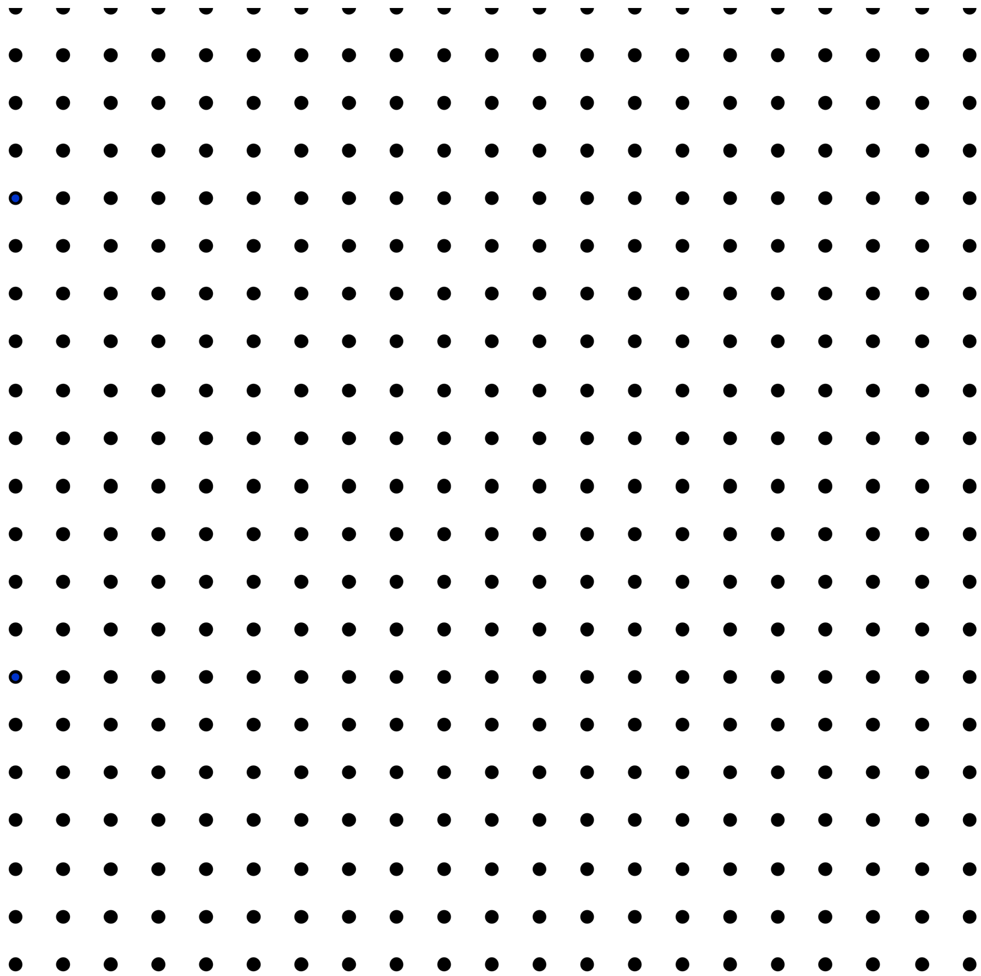
Task 4

a) In Task 3c), How are these figures in i) different from the figures in ii) ? What property distinguishes figures in i) from figures in ii)?

b) How would you modify the relation in Task 3b) such that it holds for all figures?

Task 5

Draw five more figures on the grid provided below and check if the relation holds for these figures as well. Are you sure that it will hold for any figure that you may draw? What are the properties common to the figures for which this relation holds?



Proving the relation we got!

Task 6

a) For a square of side *m* units

The number of grid-points in the interior (I) is ____________

The number of grid-points on the boundary (B)is ____________.

I + $\frac{B}{2}$ - 1 = ____________.

How is the expression I + $\frac{B}{2}$ - 1 related to the area of the square?

b) For a rectangle of length *l* units and breadth *b* units

The number of grid-points in the interior (I) = _______________

The number of grid-points on the boundary (B) = ____________

I + $\frac{B}{2}$ - 1 = ____________

How is the expression I + $\frac{B}{2}$ - 1 related to the area of the rectangle?

For a figure Q, with I grid-points in its interior and B grid-points in its boundary, let us call

$I + \frac{B}{2} - 1$ as **Pick(Q).**

Then for a rectangle and square, we saw that

Pick(Q) = Area(Q)

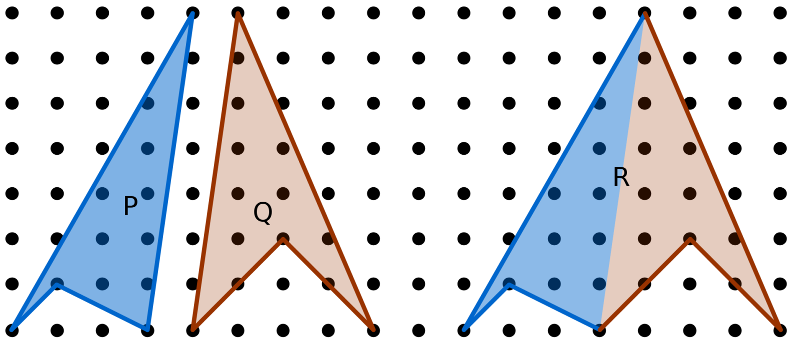
This is called Pick's Theorem.

Thus we proved Pick's theorem for a restricted class of figures namely squares and rectangles. We now will go on to see if Picks Theorem is true for all figures on the grid paper. But before that, let us go back to Task 1!

c) Solve Task 1.

Task 7

If we put together two figures, say figure P and figure Q, in such a way that they share a boundary, to form a compound figure R, the Area of (R) = Area (P) + Area (Q)



Let I_P and I_Q be the number of grid-points in the interior of P and Q respectively and B_P and B_Q be the number of grid points in the boundary of P and Q respectively.

Let is say that Pick(P) = $I_P + (B_P / 2) - 1$

Pick (Q) = $I_Q + (B_Q /2) - 1$

Since Pick (P) and Pick (Q) are the areas of P and Q respectively, and the areas of P and Q add up to give area of R, we would expect that Pick (P) and Pick (Q) would add up to give Pick (R) as well.

Now Pick (R) = $I_R + (B_R / 2) - 1$

, where I_R and B_R are the number of grid-points in the interior and boundary of R.

Now how are the number of grid-points in the interior of R related to those in the interior of P and Q?

Now how are the number of grid-points in the boundary of R related to those in the boundary of P and Q?

a) Can you come up with an expression for I_R and B_R in terms of I_P, I_Q, B_P and B_Q?

I_R = _______________

B_R = _________________

b) Substitute these in the expression for Pick (R) and verify that Pick (R) = Pick (P) + Pick (Q)

Task 8

Use the result in Task 7 to prove Pick's Theorem for more general figures. {Hint: Start with right triangles, then move to any triangle and then onto figures that can be divided into triangles.}

What properties should the figure have for Pick's Theorem to hold?

References

https://kurser.math.su.se/pluginfile.php/15491/mod_resource/content/1/picks.pdf
http://www.geometer.org/mathcircles/pick.pdf
Ian Stewart (1992) - Another Fine Math You've Got me into , Dover Publications

