

Exploring a Dot Grid Through Rectangles

Introduction

You have calculated areas and perimeters of figures like rectangles and triangles, many times. Have you ever wondered what is the relationship between area and perimeter of a figure? What happens when area increases? Does the perimeter increase or decrease? What happens to the area if the perimeter decreases?

Here we will explore the relationship between the area and perimeter of rectangles. We will do this, however, with an important constraint - the rectangles will be those that can be drawn on a dot grid such that the corners (vertices) of the rectangles are grid points.

Materials

Grid papers (each student will require three to four square dotted grid papers), pencils.

Look at the grid paper you have.

Let us call the length of the line segment AB (shown in figure 1) as one unit of length.

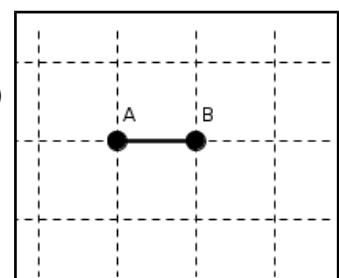


Figure 1

Now, considering the above length as one unit, what can you say about the area of this shape in figure 2? Why do you think so? Discuss with your friends.

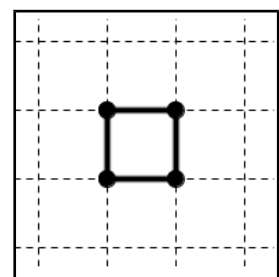


Figure 2

Task 1: Any more unit areas?

Draw a few more (at least two) figures such that their areas are also one square unit.

Task 2: When all adjacent sides are perpendicular...

The polygons drawn in figure 3 are called rectilinear polygons.

Draw five such figures and find their areas and perimeters.

Remember that all the vertices of these figures should be on the grid points.

* A rectilinear polygon is a polygon, all of whose angles are either 90° or 270° . Some examples of rectilinear shapes are given here.

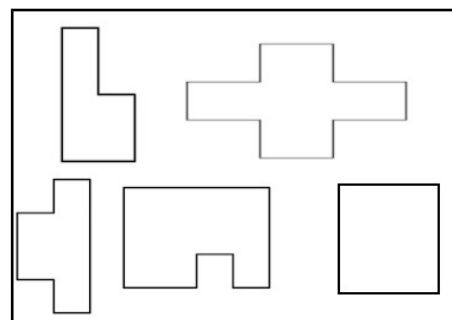


Figure 3

Task 3: What if some adjacent sides are not perpendicular...?

Figure 4 shows some non-rectilinear polygons. Find the area of the given figures.

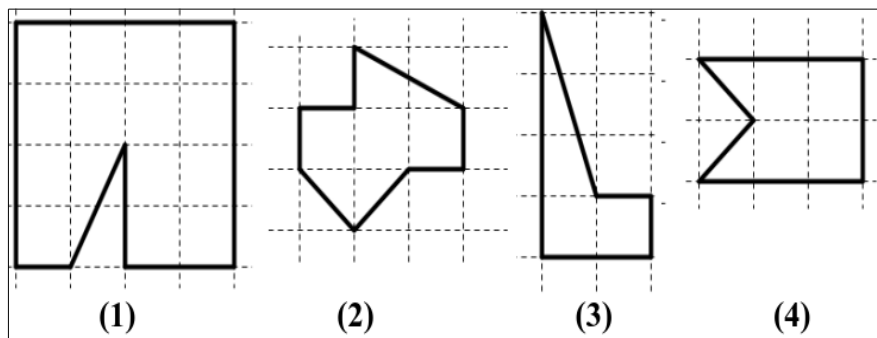


Figure 4

Task 4: Focusing on rectangles

Draw five rectangles on the grid paper. Keep in mind the following:

- 1) The vertices of the rectangles should be grid points.
- 2) The rectangles should be of different sizes.
- 3) At least one of the rectangles should be tilted.
- 4) Measure and write the area and the perimeter of the rectangles which are not tilted. Discuss how you got your answers.

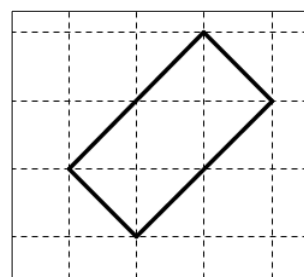


Figure 5 Tilted rectangle

The vertices of the rectangles should be grid points!

Task 5 : Perimeter same, but rectangles different

Sub task 1

- 1) Draw different rectangles on your dot grid, all of which have a perimeter of 16 units. (Draw as many as you can.)
- 2) Complete the table given below based on your rectangles.
- 3) Which rectangle has the highest area? Which rectangle has the smallest area?
- 4) Compare your table with those of your friends.
- 5) Did you get a square in your table? Is a square also a rectangle? _____

Note: "Semi-perimeter" of the rectangle is half of the perimeter.

Perimeter is 16 units				
Rectangle	Length	Breadth	Semi-perimeter	Area
i				
ii				
iv				
v				
vi				
vii				
viii				
ix				

Sub task 2

- 1) Draw different rectangles on your dot grid, all of which have a perimeter of 18 units. (Draw as many as you can.)
- 2) Complete the table given below based on your rectangles.
- 3) Which rectangle has the highest area? Which rectangle has the smallest area?
- 4) Compare your table with the tables drawn by your friends.
- 5) Did you get a square in your table?

Perimeter is 18 units				
Rectangle	Length	Breadth	Semi-perimeter	Area
i				
ii				
iii				
iv				
v				
vi				
vii				
viii				
ix				

Perimeter is ___ units				
Rectangle	Length	Breadth	Semi-perimeter	Area
i				
ii				
iii				
iv				
v				
vi				
vii				
viii				

The vertices of the rectangles should be grid points!

Task 6: Now... some same area rectangles**Sub task 1**

- 1) Draw different rectangles which have an area of 36 square units. (Draw as many as you can).
- 2) Fill the given table.
- 3) Did you get a square?
- 4) Compare your table with the tables drawn by some of your friends.

Area is 36 square units				
Rectangle	Length	Breadth	Semi-perimeter	Area
i				
ii				
iii				
iv				
v				
vi				
vii				
viii				
ix				

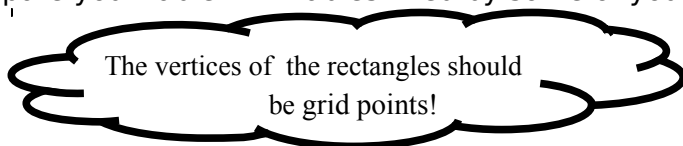
Sub task 2

- 1) Draw different rectangles which have an area of 17 square units. (Draw as many as you can).
- 2) Fill the given table.
- 3) Do you see any congruent rectangles among the rectangle you have drawn?
- 4) Compare your table with the tables filled by some of your friends.

Area is 17 square units				
Rectangle	Length	Breadth	Semi-perimeter	Area
i				
ii				
iii				
iv				
v				

Sub task 3

- 1) Draw different rectangles having an area of 24 square units. (Draw as many as you can).
- 2) Fill the given table.
- 3) Do you see any congruent rectangles among the rectangle you have drawn?
- 4) Compare your table with tables filled by some of your friends.



Task 7 : Next, rectangles with equal areas and equal perimeters

Can you draw two different rectangles whose perimeter is 14 units and area is 12 area units? How many different rectangles did you get? Compare your rectangles with those drawn by your friends. Are they the same?

Did you get different rectangles? If yes, share your answer with your friends and your teacher. If you think it is not possible, think about why not?

Task 8: More with the grid: possibilities and impossibilities

Remember that all the vertices of all the figures you draw should be grid points.

Use the grid paper, explore, and find answers to the following:

1) If the length and the breadth of a rectangle are natural numbers, and its area is an odd number, what can you say about that rectangle's semi-perimeter (half of the perimeter)?

2) If the length and the breadth of a rectangle are natural numbers, and its semi-perimeter is an odd number, what can you say about the area of this rectangle?

3) What are the different possible areas of triangles drawn on the grid? Are all the multiples of half achieved?

4) What are the possible areas of grid squares (squares with vertices on the grid)? (These may not have integer sides!)

References

- De, P., Sircar, S., & Titus, S. (2017, November). LFHC - Area, Perimeter and Congruency (APC). *At Right Angles*, Volume 6, No.3, pp.53-58. Azim Premji Foundation.
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