UNDERSTANDING OPEN EXPLORATION IN A CLASSROOM

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In the present paper, we have focused on the 'pedagogical openness' of an activity that has a large number of possibilities and enables applications of a wide range of mathematical concepts. Apart from that the 'openness' also allows teachers to design the course of the activity depending on their own beliefs of mathematics and mathematics teaching. The approaches that two teachers took were very different with respect to the mathematical processes encouraged in their classrooms. In one classroom, the teacher had an approach which encouraged the processes of proving the patterns that the students had found and in the other classroom teacher encouraged the students to make more and more conjectures. Open exploration tasks offer an openness in the class where the classroom discourse depends on student-teacher relations, student preparation and interests and teachers' own beliefs about mathematics and mathematics teaching.

Keywords: Teacher beliefs, Open Exploration, Mathematical processes

INTRODUCTION

Teachers' beliefs about mathematics and mathematics teaching and their own ideas of effective mathematics teaching is associated with teachers' classroom practices *(Fennema et al., 1996)* and their willingness to engage in pedagogies centered around students *(Heck, Banilower, Weiss, & Rosenberg, 2008)*. *Ernest (1991)* argued that a mathematics teacher's belief system has three parts; the teacher's ideas of mathematics, of teaching of mathematics, and of learning of mathematics. "Discovery-oriented teachers view mathematics as a set of knowledge best learned through student-guided exploration" *(Polly, D. et al., 2013)*.

In this paper, we look at an activity, aimed at encouraging mathematical processes through student explorations. The activity was conducted with two different sets of children by two different teacher-researchers. The way the two teachers designed the course of the classrooms were indicators of the teachers' own beliefs of mathematics and mathematics education. In this paper, we argue that open exploration tasks allow teachers to design their classrooms based on their ideas of mathematics, the nature of mathematics learning and what according to them entitles learning of mathematics.

ABOUT THE STUDY

Students need to have opportunities where they evolve themselves mathematically and engage in tasks which allow them to explore mathematical ideas. Moreover, they make sense of the knowledge that arises from the collective discussion of the tasks *(NCTM, 2000)*. Teaching such open exploratory task needs preparation for teaching, which includes teachers' beliefs on what is important that a student should learn, and what it means to do meaningful mathematics.

Here we look at two mathematical classrooms which focused on exploring students' thinking while engaging in an open exploration of patterns. These classrooms were a part of a larger talent nurture programme, which is aimed at supporting high quality and well-rounded science and mathematics learning. We aim to understand how mathematically enriched tasks can give teachers the freedom to guide the students, based on their own experience, beliefs and their understanding of students' engagements and preparations.

METHODOLOGY

The data was collected from two classrooms where the same mathematical exploration task was conducted. These classrooms were a part of a summer school held for students from 7 different English medium schools around the institute, These schools cater to students from a mixed socioeconomic background. All the students were Class 10 students (entering). The admission to the summer school was completely voluntary and there was no selection process. Classroom 1 had 22 students (B – 12 and G – 10) and Classroom 2 had 25 students (B – 14 and G – 11). Data sources include classroom observations, log files, and classroom videos. The two classrooms were taught by two different researchers. One of the teacher has a doctorate in pure mathematics and has minimal experience in actual classroom teaching. The other teacher has a masters degree in mathematics and has taught in schools for at least 5 years. Both of them have been mathematics educators for more than a decade. Detailed interactions with teacher will be added in the full paper.

The objective of the task was to encourage different mathematical processes in the classroom. In the tasks, students explored patterns of squares of natural numbers.

ABOUT THE ACTIVITY

The activity comprised of two different but connected tasks. In the first task, students were given the following table (Table 1.1) and were asked to observe patterns in it.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

Table 1.1

In the second task, natural numbers up to 400 were arranged in an 8-column table as shown below (Table 1.2) and the first few square numbers highlighted. They were expected to shade in the remaining squares and look for patterns.

I	II	III	IV	V	VI	VII	VIII		I	II	III	IV	V	VI	VII	VIII
1	2	3	4	5	6	7	8		209	210	211	212	213	214	215	216
9	10	11	12	13	14	15	16		217	218	219	220	221	222	223	224
17	18	19	20	21	22	23	24		225	226	227	228	229	230	231	232
25	26	27	28	29	30	31	32		233	234	235	236	237	238	239	240
33	34	35	36	37	38	39	40		241	242	243	244	245	246	247	248
41	42	43	44	45	46	47	48		249	250	251	252	253	254	255	256
49	50	51	52	53	54	55	56		257	258	259	260	261	262	263	264
57	58	59	60	61	62	63	64		265	266	267	268	269	270	271	272
65	66	67	68	69	70	71	72		273	274	275	276	277	278	279	280
73	74	75	76	77	78	79	80		281	282	283	284	285	286	287	288

Table 1.2: Snapshot of the entire of table

Classroom 1

In Classroom 1, the students were asked to find patterns from table 1.1. They were given time to work individually or in groups. Once they had figured out some patterns, the teacher asked them to share the patterns with the whole class. A norm was set such that if any pattern was not clear, students could ask questions about it to the student who had thought of the pattern. Following is an instance from the classroom.

St 1: If you multiply a number and it's consecutive number and then add the consecutive number you will always get square of the consecutive number. [To which teacher asked the whole class]

T1: Do you think this always correct? [class in coherent *yes*] So if I take an example of 1027 *1028 and add 1028 will get the square of 1028? [Class in coherent Yes ma'am] [Teacher wrote the same on the board and asked the class] and I will the square? Really? [looking at the student sitting on the last bench] What do you think St 2?

St 2: It could.

T1: It could, so it might not be? So what does one do when this happens? [students discussing in a group, T continue] St 2 is saying it might work, it might not work. So, in this situation what does one do?

St 3: Make it a Theorem?

T: [repeated] Make it a theorem, so how do you make something a theorem St 4?

St 4: By proving it.

T: Correct, so you got a lot of theorems here [pointing at the board where students patterns are compiled], so why don't we start by the first one.

Here in the above instance, we can see that the need for proving the statements have come from the students and not from the teacher. But the belief of the teacher that 'doing' mathematics means proving statements very much evident from the way the class was guided towards it. And the main concern of the teacher was to arrange students' statements and to promote the clarification of the emerging mathematical ideas. The teacher wanted to take advantage of the discussion on proof and theorems to reinforce the importance of proving the conjectures.

Classroom 2

In this class, the teacher has started the class by the introducing of herself and asking the students if they find the activities in the camp any different from their usual learning experiences. Students replied that there were many practicals and they enjoyed it as it was not done in the traditional school teaching methods. The teacher emphasized on that point and announced to the entire class that today also you will be doing something different from the school. She told them that "You will be making mathematics" and added "What does it mean by making mathematics? This indicates the teacher's inclination towards children making their mathematics. In this class also the tasks conducted were the same. Following is an instance from the classroom.

St 1: The numbers between the square numbers are increasing by 2 [Teacher wrote this on board and marked as St 1's pattern].

T 2: [repeated the statement] what does that mean?

St 1: Between 1 and 4, it is 2 and 3. Between 4 and 9 it is 5,6,7,8.

T 2: How will I know what you are saying is correct? I take any big square no how will I know how many numbers are going be there in between? [….]

St 2: St 1's pattern is proven if we write all the numbers between the two squares from 1 to 20 we can see that the numbers in between are 2, 4, 6, 8 and so on.

St 3: That will not be the proof of St 1's pattern that will be just the verification of St 1's pattern.

Later in the class, St 3 proved St 1's pattern and it was written on the board by the teacher that St 1's pattern is proved and verified. Here the teacher has given importance to the patterns emerging from the students' discourse. This was evident in the discussion also. While students were sharing their views they tend to say that " I disagree with St X's pattern", or " I have a counterexample to St Y's Pattern".More importantly the focus of the class was more on finding as many patterns as they could from the table that was given to them. Later the teacher decided to go ahead with the other task (Table 1.2) instead of spending time on proofs as she felt that the students were more interested in finding more patterns rather than proving it. But she made sure that students develop an understanding of the difference between what has been proven and what has been just stated.

CONCLUSIONS

In both the classes students reported that they enjoyed finding patterns. They noted that in the given task there were more than one patterns. In both the instances, we saw two researchers conduct the same tasks but the way it progressed was different. In Classroom 1, the teacher emphasized more on students proving the patterns that had emerged from the class, while in Classroom 2, the teacher's emphasis was more on finding more patterns in different contexts. In Classroom 1, the teacher emphasized more on students engaging with the processes of conjecturing to proving for every pattern that was suggested while in the Classroom 2, the teacher encouraged children to come up with more and more patterns and make their own mathematics. The way the two classes progressed indicate differences in the teachers' own beliefs of what is mathematics and how to engage students in mathematical processes. Open mathematical explorations of the type conducted in the classrooms not only give opportunities to students to engage with mathematics and mathematical processes but also give teachers opportunities to structure or design the classroom based on their own beliefs.

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