Mid-Point Quadrilaterals

Introduction:

In this learning unit, you are going to use a software app called GeoGebra to explore the quadrilaterals formed by joining the midpoints of any given quadrilateral. You are also going to make many conjectures about special cases of these quadrilaterals using GeoGebra and are going to try and prove the conjectures.

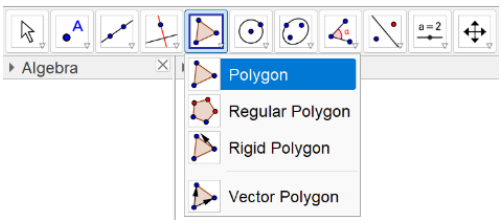
Sketch and Investigate

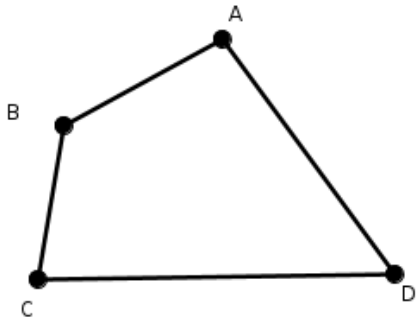
Open a new page in GeoGebra. Click on View and then on Axes to hide the axes. Only the algebra and graphics views should be visible.

Task 1

Construct a quadrilateral ABCD using the Polygon Tool.

To do this click on the **Polygon tool** and click the points in the following order: point *A*, point *B*, point *C*, point *D*, and then point *A* to close the polygon.

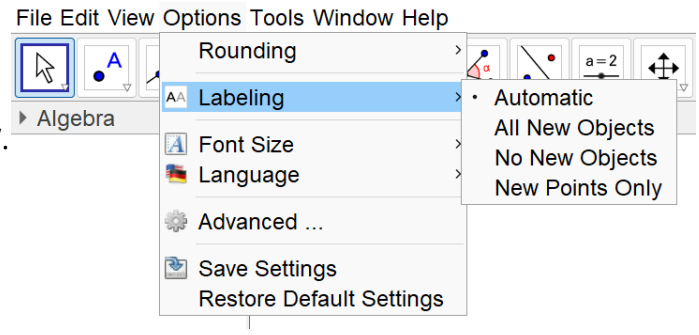




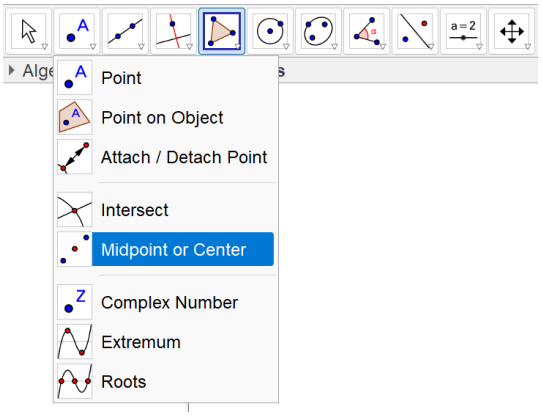
Did you notice that GeoGebra labels the vertices with uppercase letters and the line segments with small case letters. This is a common convention.

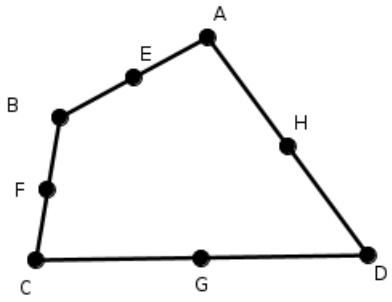
Right-click on each of the sides of the quadrilateral ABCD and unselect Show Label from the menu to hide the labels a, b, c, and d of the sides.

To avoid cluttering the figure with too many labels, go to Options in the main menu, and select 'Labeling' followed by 'New Points Only'. This will ensure that new line segments are not labeled.



Find the midpoints of the sides, AB, BC, CD, and AD. Some of them are marked here. This can be done by selecting the Midpoint or Center icon from the Point Tool menu and then clicking on the four sides of the quadrilateral ABCD.



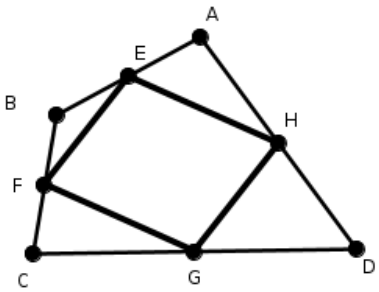


The midpoints will be labeled as E, F G, and H respectively.

Make the new quadrilateral by selecting the Polygon tool and then clicking on the points E, F, G, and H in that order. This will be called the midpoint quadrilateral.

By a mid-point quadrilateral, we mean a quadrilateral formed by joining all the midpoints of the sides of a given quadrilateral.

Drag the vertices of the original quadrilateral ABCD and observe what happens to the midpoint quadrilateral EFGH. Record your observations below.



Is there any similarity between all the mid-point quadrilaterals you got while dragging the vertices?

Note the lengths of the four sides of quadrilateral EFGH (which are named as e, f, g, and h) in the Algebra view. What do you observe? Now drag any one of the vertices of the original quadrilateral ABCD. What are your observations regarding e, f, g, and h? Based on the observations, what can you say about them?

What kind of quadrilateral do you think EFGH is? Use your observations to support your conjecture.

Would you be able to make similar conjectures by considering the diagonal BD of quadrilateral ABCD instead of diagonal AC?

Task 2

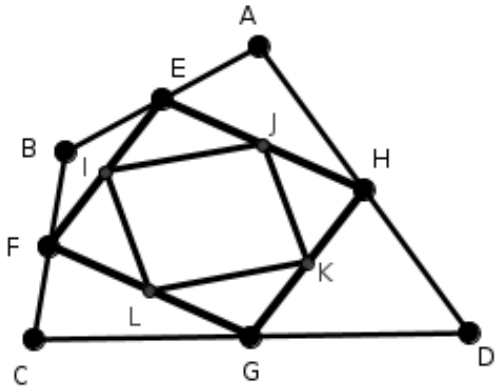
Prove your conjecture.

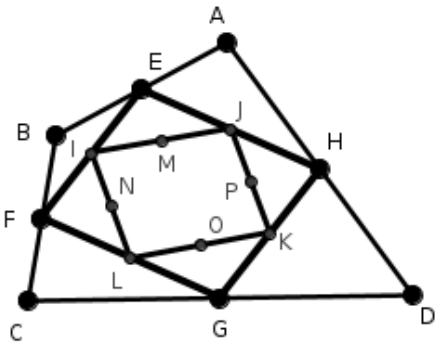
Task 3

Observe the numbers associated with Poly1 (ABCD) and Poly2 (EFGH) in the Algebra view. Can you see a relationship between them?

From this, what can you say about the area of the midpoint quadrilateral EFGH in relation to the area of ABCD?

Draw the midpoint quadrilateral of EFGH. GeoGebra will label it as IJKL. What kind of a quadrilateral is IJKL? How is its area related to that of EFGH and ABCD?





Continue drawing midpoint quadrilaterals as you did in Task 1. Can you see how these midpoint quadrilaterals are related to each other?

Task 4

Prove your conjecture.

Task 5

Using GeoGebra, draw a rectangle. Move the vertices of the rectangle you have drawn and check if the rectangle remains a rectangle. If not, draw again.

Write down the steps you took to ensure that you have drawn a rectangle.

Draw a mid-point quadrilateral of this rectangle. What can you say about this mid-point quadrilateral? Can you prove your conjecture?

Task 6

Using GeoGebra, draw a rhombus. Move the vertices of the rhombus you have drawn and check if the rhombus remains a rhombus. If not draw again.

Write down the steps you took to ensure that you have drawn a rhombus.

Draw a mid-point quadrilateral of this rhombus. What can you say about this mid-point quadrilateral? Can you prove your conjecture?

Task 7

Using GeoGebra, draw a square. Move the vertices of the square you have drawn and check if the square remains a square. If not draw again.

Write down the steps you took to ensure that you have drawn a square.

Draw a mid-point quadrilateral of this square. What can you say about this mid-point quadrilateral? Can you prove your conjecture?

References:

Lingefjärd, T., Ghosh, J, Kanhere, A. (2015). Students Solving Investigatory Problems with GeoGebra - A Study of Students' Work in India and Sweden. In S.J. Cho (Ed.),
The Proceedings of the 12th International Congress on Mathematical Education