Finding the Right Path

Task 1: Seven Bridges of Konigsberg!

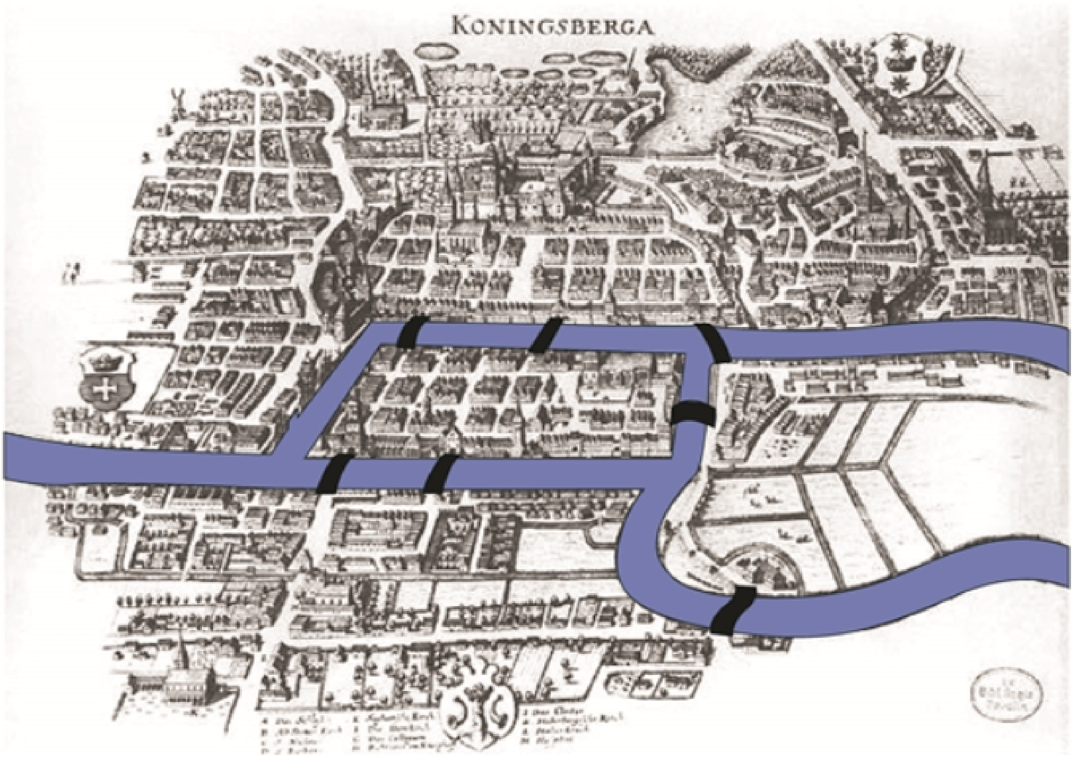

Figure 1

Today we are going to begin with the story of Konigsberg in the 18th century, its geography, bridges, and the question asked by its citizens.

Kaliningrad is a city which lies between Lithuania and Poland and is at some distance from the rest of Russia. In fact, it was originally a German town and was called Konigsberg. The river that runs through this town was then called the river Pregel. The Pregel branched and looped through Konigsberg, as shown in the picture, and in the eighteenth century there were seven bridges across it.

A challenge took shape around the river and the bridges. Is there a route that would let one walk across all seven bridges exactly once? No bridge could be missed or crossed twice and, of course, there was to be no swimming across the river!

Q.1. Can you state the problem of walking over the 7-bridges in your own words?

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Look at the following picture.

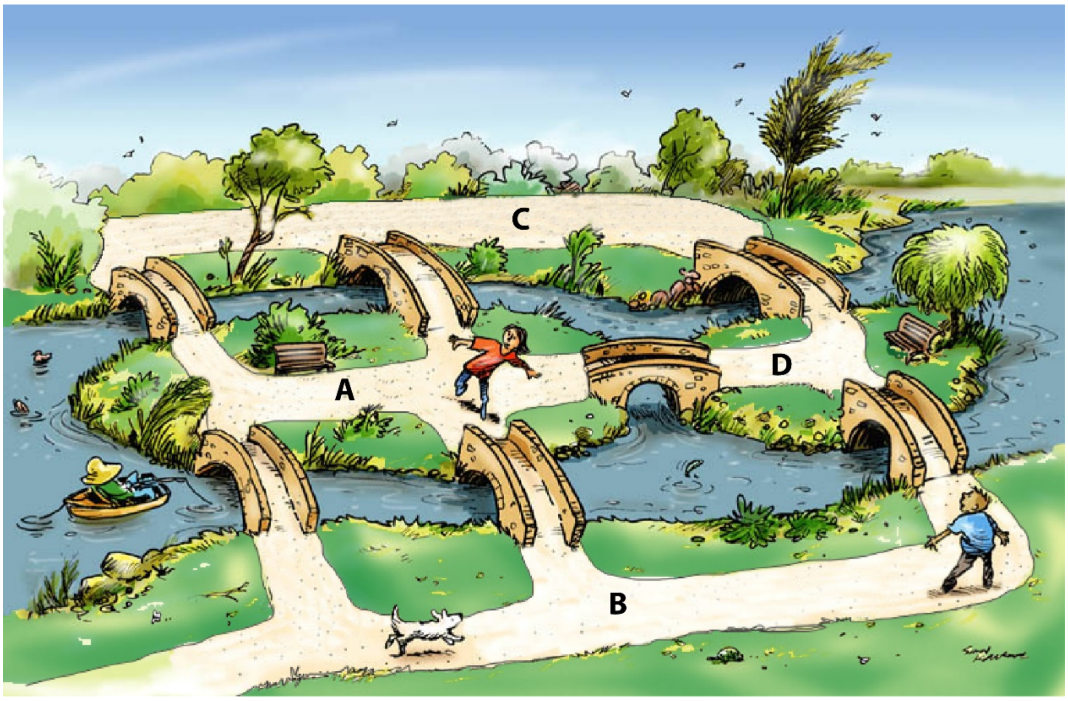

Figure 2

Look at the picture given above. *A*, *B*, *C*, and *D* represent land-masses or islands and *a*, *b*, *c*, *d*, *e*, *f* and *g* represent bridges.(Bridges are not marked in the given figure) Suppose you are asked to find a path which covers all bridges but crossing each bridge exactly once while missing no bridge.

Q.2. Can you find the required path? Share your paths with your friends.

Q.3. Is this picture same as the one you saw of Konigsberg Bridges? Why do you think so?

Think about further simplifying this picture. Remove the details not required to solve the problem? Draw your simplified pictures here, and discuss with your partner how your picture/diagram still represents the problem of 7-bridges of Konigsberg.

Do you remember the popular childhood puzzle about drawing a square and its diagonals without lifting the pencil or retracing any part?

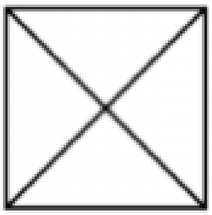

Figure 3

Q.4. Were you successful? How did you do it?

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Task 2: Graphing the Reality!

Q. 5. See the following graph. This graph represents the same 7-bridges problem that we were working on till now. Explain how it is the same problem. Where are the bridges and lands?

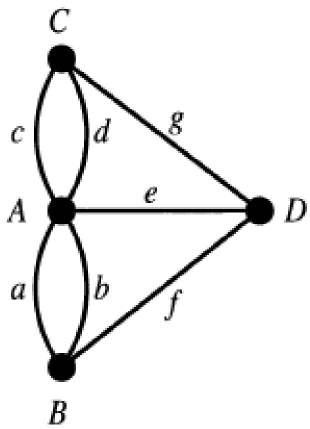

Figure 4

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Can you trace the entire graph above without lifting your hand? Now this problem is same as the problem citizens of Konigsberg came across – waking over all the bridges once. Try here, and try with different starting points.

A graph in Graph Theory consists of edges and vertices. The graphs are diagrams where there are vertices and lines joining any two vertices are called edges.

The number of edges, that join at a vertex are called as degree of the vertex.

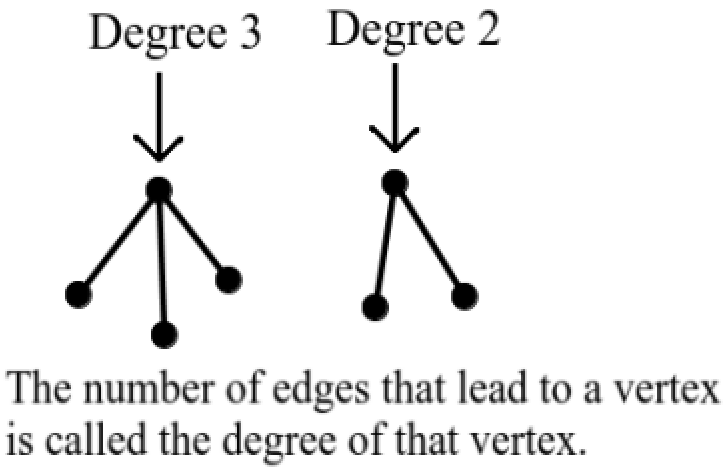

Figure 5

For example, the following diagram has 6 vertices and 7 edges, vertices A, C and E have degree 3; vertices D and F have degree 2 and vertex B has degree 1.

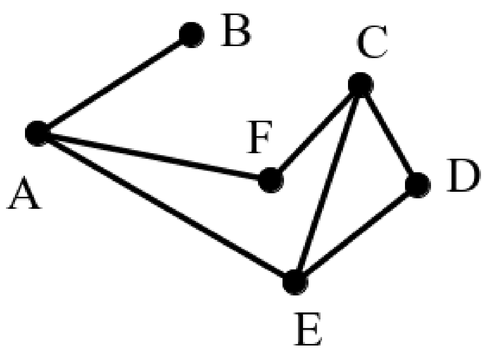

Figure 6

Draw a graph of your own; describe degree of its vertices.

Study the following graphs. In each of them, see whether it is possible to find a path that passes through every edge without repetition. Try different vertex as starting points. Describe the path as sequence of letters. If you think there is no path, write no in path column. Record your findings for each graph in the following table:

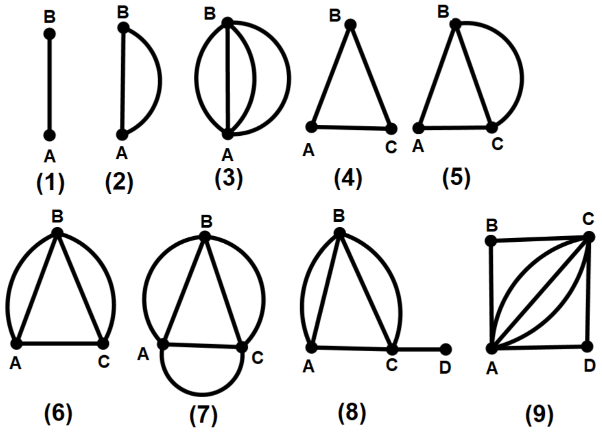

Figure 7

Record your findings for each graph in the following table:

Graph No.	Path	Are initial vertex and last vertex same? (Y/N)	Degree of initial vertex	Degree of last vertex	Degrees of other vertices
1					
2					
3					
4					
5					
6					
7					
8					
9					

Q. 6. Study the pattern carefully in the table and write your guesses about what features of the graph makes the graph traceable (crossing each edge only once) without lifting your hand. Also, if you think that some features make them non traceable, what are they?

Q.7. What pattern do you see for the graphs where the starting and ending point of the path is the same vertex? Write statements of your conjectures.

Q.8. What pattern do you see for the graphs where there is no path?

Q.9. How do you know the statements you made are true?

Do the same exercise for the following set of graphs.

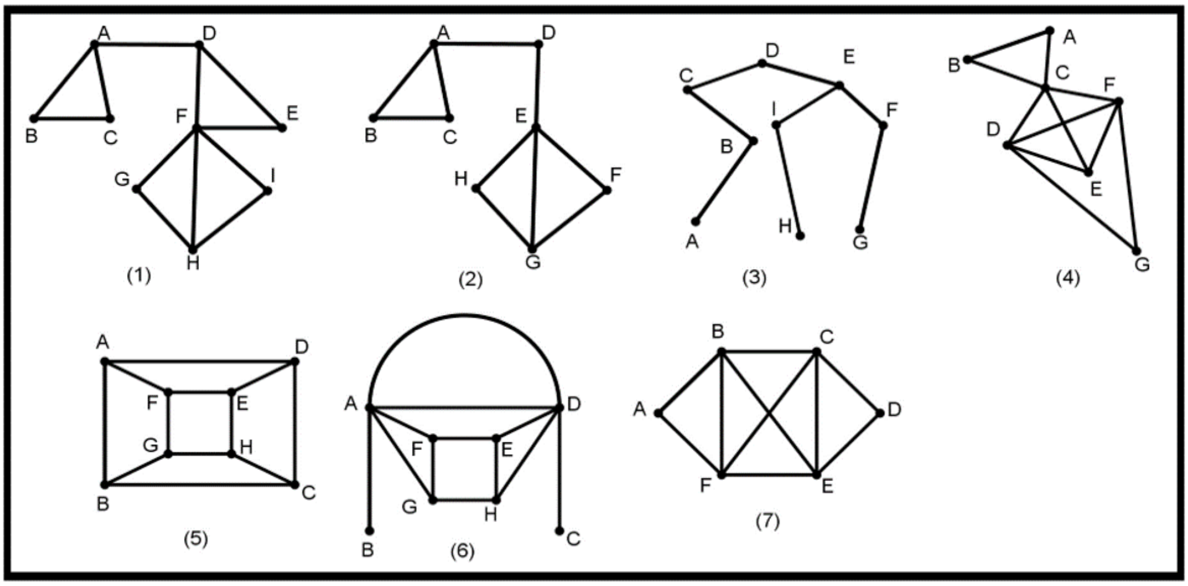

Figure 8

Were you able to find a path in all the graphs given above?
Discuss and record your findings for each graph in the following table. If you think there is no path, write no in path column.

Graph No.	Path	Are initial vertex and last vertex same? (Y/N)	Degree of initial vertex	Degree of last vertex	Degrees of other vertices
1					
2					
3					
4					
5					
6					
7					

Q.10. Study the pattern carefully in the table and write your guesses about what features of the graph makes the graph traceable (crossing each edge only once) without lifting your hand.

Q.11. What pattern do you see for the graphs where the starting and ending point of the path is the same vertex? Write statements of your conjectures.

Q.12. How do you know the statements you made are true?

Coming back to Konigsberg. The citizens of Konigsberg had a hard time solving this problem. Their mayor wrote to the famous mathematician Leonhard Euler for help. And the first thing Euler did was to create a simplified and labeled drawing. Here is his drawing:

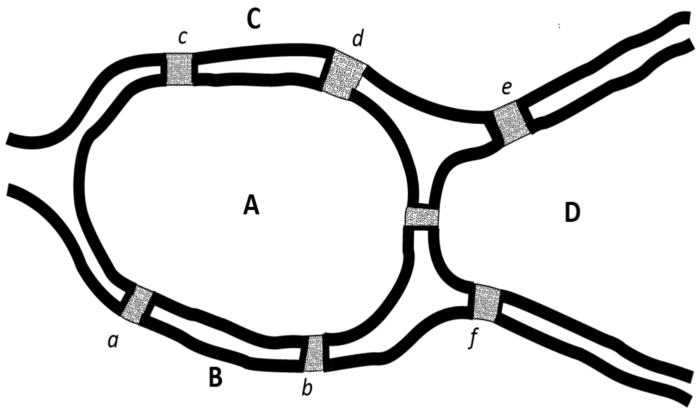

Figure 9

Q. 13. Look at this map that Euler made. How do you make sense of it? What are those letters? Compare it with the map of Konigsberg and its bridges. Can you make a graph out of this diagram?