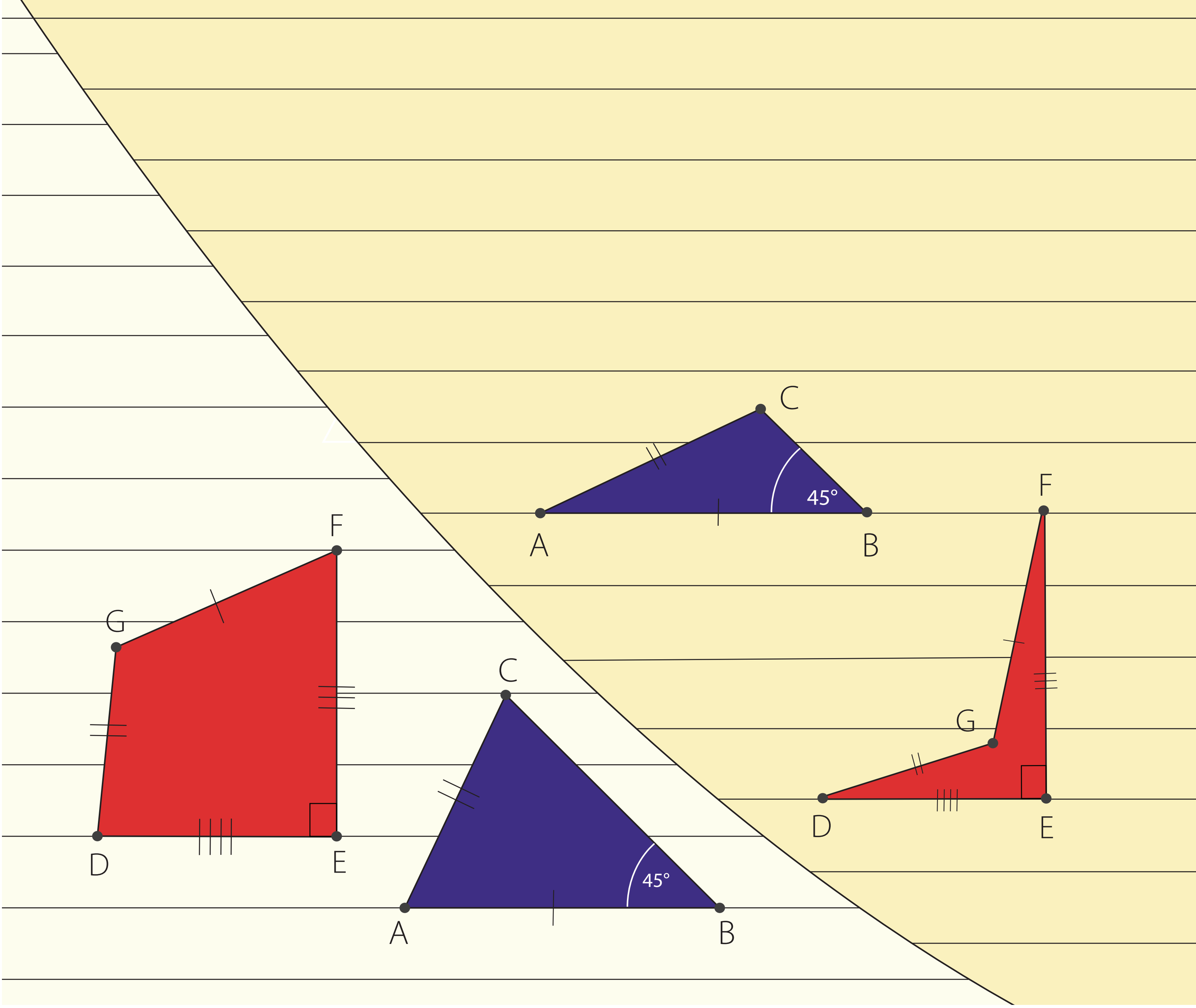

8.14

Is your polygon the same as mine?

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8.14. Is your polygon the same as mine?

Introduction

Have you ever wondered how you would describe a triangle that is in your mind to somebody over the phone? What do you really say? Do you mention the sides or the angles? And would that person get the exact same figure that you had in mind? Moreover, how can you do this by giving minimum information? Today we will try to answer these questions by investigating some examples, making observations, and verifying or refuting these observations.

Materials: Blank sheets, pencils, erasers, Geometry boxes (Geometry compass, set-squares, protractor, and scale), scissors.

Task 1: Drawing your triangle

Q1. Draw a triangle of your choice on the given blank sheet of paper. Measure the sides and the angles of the triangle, and label the vertices of the triangle.

Q2. Now see the triangles drawn by your friends. Do you see anything interesting? What is it?

_ _

_ _

Keep the paper on which you drew your triangle safely aside; we will be coming back to this triangle later in the activity.

Task 2a: Constructing a triangle when only one side is given

Draw a triangle on the given paper, such that one of its sides is 6 cm. Label the vertices of your triangle.

Now study the triangle drawn by your partner.

Q1. Is your triangle the same as your partner's?

_ _

Q2. How did you compare these two triangles?

_ _

Q3. You and your partner, both were told that one side of the triangle is 6 cm. Did you both get exactly the same triangles? Why?

_ _

Task 2b: Constructing a triangle when only one angle is given

Draw a triangle on the given paper where one of the angles measures 55°. Name your triangle.

Now study the triangle drawn by your partner.

Q1. Is your triangle the same as your partner's?

_ _

Q2. How did you compare these two triangles?

_ _

Q3. You and your partner, both were given one angle of 55°. Did you both get exactly the same triangles? Why?

_ _

Task 3: Constructing a triangle when two measures are given

Make three groups among yourselves. If possible, form your group with your classmates who are sitting close to you. Let us call these groups A, B and C.

Group A: Draw a triangle whose sides are 7 cm and 5 cm. Label the vertices of your triangle.

Group B: Draw a triangle whose one side is 6 cm and one angle is 55°. Label the vertices of your triangle.

Group C: Draw a triangle whose two angles are 50° and 75°. Label the vertices of your triangle

Now study the triangle drawn by your partner.

Q1. Is your triangle the same as your partner's?

_ _

Q2. How did you compare these two triangles?

_ _

Groups A, B and C:

Q3. Did each of you get exactly the same triangles as the members in your group ? Why?

_ _

Task 4: Constructing a triangle when three measures are given

Your class is already divided into 3 groups. Now divide each group into 2 sub-groups. A1 & A2, B1 & B2 and C1 & C2..

Group A1: Draw a triangle XYZ such that XY = 4 cm, YZ = 6 cm, and XZ = 7 cm.

Group A2: Draw a triangle ABC such that, AB = 5 cm, BC = 6 cm, and $\angle ACB = 45°$.

Group B1: Draw a triangle IJK such that $\angle IJK = 40°$, $\angle JKI = 65°$, and $\angle IKJ = 75°$.

Group B2: Draw a triangle STU such that $\angle UST = 50°$, ST = 3 cm, and $\angle STU = 65°$.

Group C1: Draw a triangle EFG such that EF = 7 cm, FG = 9 cm, and $\angle GEF = 90°$.

Group C2: Draw a triangle PQR such that PQ = 5 cm, $\angle PQR = 50°$, and QR = 4 cm.

Now study the triangle drawn by your partner.

Q1. Is your triangle the same as your partner's?

____

____

Q2. How did you compare these two triangles?

____

____

Task 5: Minimum conditions for the construction of a unique triangle

Q1. If you want your friend/partner to construct exactly the same triangle like the one you drew in Task 1, what minimum information will you have to provide?

____

____

____

Q2. In the previous question, is there a different set of information that could be provided to construct the exact same triangle? Try and mention all such different sets of information that would work.

____

____

____

Task 6: Constructing a quadrilateral

Q1a. Now that you all know how to make a congruent triangle, let us figure out how to make a congruent quadrilateral. So if the minimum conditions for making a congruent triangle are three, what should be enough for a quadrilateral?

____

____

Q1b. Now, given that all the sides of a quadrilateral are 3 cm, think about all the different quadrilaterals that you can draw. Draw the figures on the given blank sheet.

Q2. Did you or your partner get different quadrilaterals for Q1b?

____

Q3. So, if only the sides are given, is it always possible to get different quadrilaterals? How do you know?

____

Q4. Imagine that you have to write to your friend about a quadrilateral. Now think of the minimum information that you can send him/her, such that he/she gets the exact same quadrilateral as the one you had in your mind. What information will you send?

Check whether what you suggested as the minimum information really works. Try drawing different quadrilaterals for the information you said you would give your friend in the question above.

Q5. Think about why this set of information will lead to congruent or non-congruent quadrilaterals.

Q6. List the conditions that worked for constructing a unique quadrilateral.

Task 7: Some special triangles and quadrilaterals

We have found out the minimum information needed to draw congruent triangles and congruent quadrilaterals, but let us look at some special triangles and quadrilaterals and find out the minimum information we need to construct these.

Q1. How many conditions do you need to construct congruent equilateral triangles?

Q2. How many pieces of information do you need to construct congruent squares?

Q3. How many pieces of information do you need to construct congruent rectangles?

Q4. How many pieces of information do you need to construct congruent rhombuses?

Q5. How many pieces of information do you need to construct congruent parallelograms?

____

____

____

Q6. How many pieces of information do you need to construct congruent trapeziums?

____

Task 8: Constructing a pentagon

Q1. Now that you all know what conditions give constructions of congruent triangles or congruent quadrilaterals, let us figure out how to construct congruent pentagons. So, if the minimum conditions for making congruent triangles are three, and that for congruent quadrilaterals are five, what do you think is the number of minimum conditions needed to construct congruent pentagons?

____

____

Q2. Imagine that you have to write to your friend about a pentagon. Now think of the minimum information that you can send him/her, such that he/she gets the exact same pentagon as the one you had in your mind. What information you will send?

____

____

Check whether what you suggested as the minimum information really works. Try drawing different pentagons for the information you said you would give your friend in the question above.

Q3. Think about why this set of information will lead to congruent or non-congruent pentagons.

____

____

Q4. List the conditions that worked for making a unique pentagon.

____

____

Task 9: Finding the number of conditions to construct a congruent polygon

Now that you know the minimum conditions needed for constructing congruent triangles, congruent quadrilaterals and congruent pentagons, let us explore how many conditions are needed for constructing congruent hexagons, or congruent heptagons.

Make some guesses, and make constructions on the given sheets of paper. Record your guesses in table 1 below.

Number of sides in the polygon	Name of the polygon	Minimum conditions required for constructing a congruent polygon
3	Triangle	3
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	

Table 1 *Minimum conditions required for constructing a congruent polygon*

Proving our Conjectures

Let us find out how can we prove which guesses are right and which ones are wrong.

Draw a quadrilateral.

Draw a diagonal inside the quadrilateral so that it splits into two triangles.

See Figure 1.

(Here we have drawn two different types of quadrilaterals).

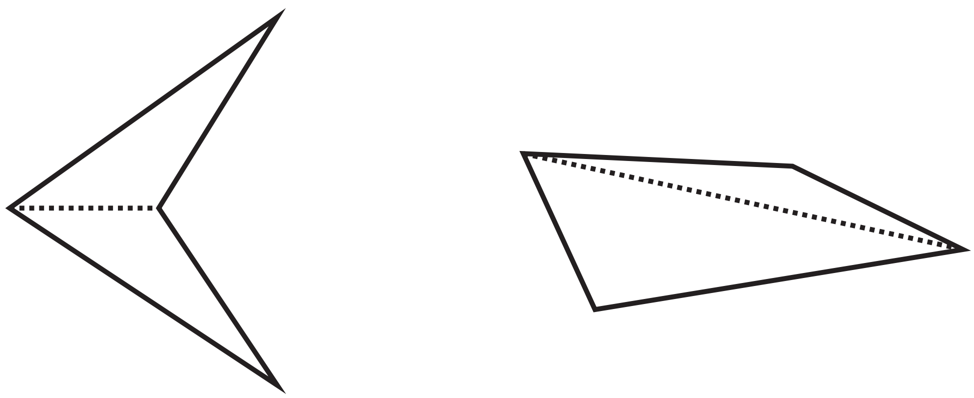

Figure 1 *Triangulation of quadrilaterals*

We see that every quadrilateral can be split into two triangles in this way. We know that for constructing a unique triangle we need three minimum conditions.

So in this case, to construct the first triangle we needed three minimum conditions. For the next triangle, we need three more, but as one side overlaps, we need only two conditions to construct a triangle congruent to the second triangle. These can be, for example, one side and the angle it makes with the adjacent side of the quadrilateral. Alternatively, one can also give two angles.

Another way of thinking about this is, once we fix the first triangle, three vertices of the quadrilateral are fixed. So to fix the remaining vertex, two conditions (as in the examples above) are sufficient. Hence these five conditions are the minimum pieces of information needed to construct a quadrilateral.

This also reconfirms our understanding of the minimum conditions needed to construct a unique quadrilateral.

What will happen if we do the same for a pentagon?

Let us draw a pentagon and see how many triangles the pentagon can be split into by drawing a minimum number of diagonals. We see from figure 2 that by drawing two diagonals, the pentagon can be split into three triangles.

For the first triangle we need three conditions, for the second triangle we need another three, but then one side overlaps so we need only two. Similarly, for the third triangle, we need two more conditions.

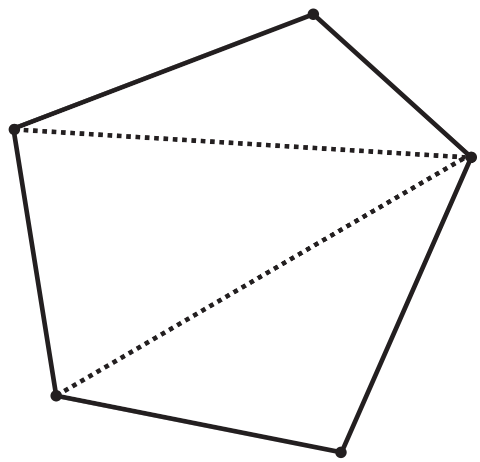

Figure 2

So, you can see that whenever you add a triangle, you add two conditions. So, the minimum conditions necessary for constructing a unique pentagon are seven (3 + 2 + 2).

Let us try to figure this out for hexagons, heptagons, and octagons.

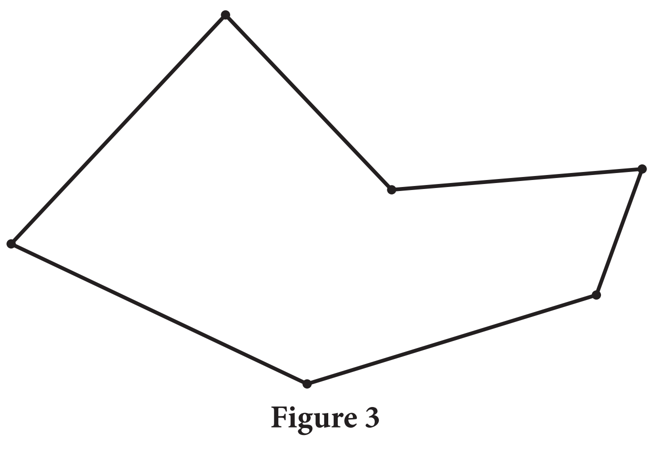

Figure 3

1. How many triangles can a hexagon be split into? (Remember that the number of diagonals drawn must be a minimum.) ____
2. What is the minimum number of conditions needed to construct unique hexagons? Why?
   ____
3. How many triangles can a heptagon be split into? ___________________________

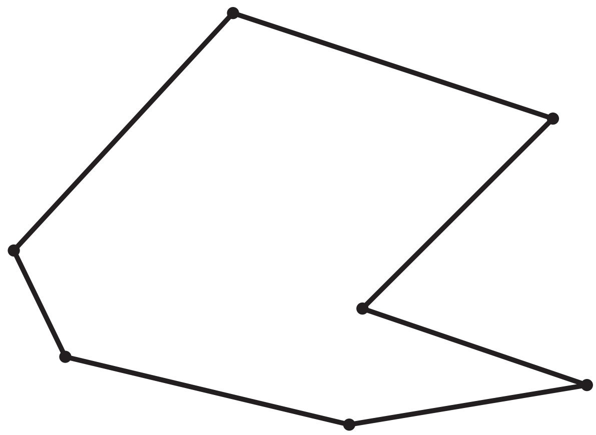

Figure 4

4. What is the minimum number of conditions needed to construct unique heptagon? Why?
   ____

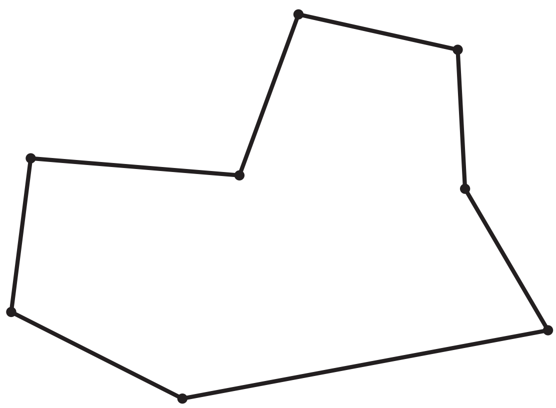

Figure 5

5. How many triangles can an octagon be split into?_____________
6. What is the minimum number of conditions needed to construct congruent octagon? Why?

____

References

- Plaksin, O., & Patkin, D. (2016). Exploring the minimum conditions for congruency of polygons. *Learning and Teaching Mathematics,* 2016(2), 42-46. http://www.amesa.org.za/amesal_n21_a12.pdf
- Page, John D. *Congruent polygons*. From Math Open Reference. https://www.mathopenref.com/congruentpolygons.html