Teachers’ conceptions of proofs while engaging in an open exploration

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Mathematics disciplinary practices involve students practising mathematics themselves, and thus teachers are anticipated to support students in developing these skills. In this work, we examine how teachers themselves deal with open mathematical explorations - which expects them to engage in mathematical processes of finding patterns, verifying them, and proving or disproving them. For this, we analysed teachers’ assignments on a Learning Unit of Vigyan Pratibha project called ‘Exploring patterns in square numbers’. This unit was conducted virtually as a part of Vigyan Pratibha Discussion Sessions (VPDS) and the participants were science and mathematics teachers.

Vigyan Pratibha (VP) is a national programme for teacher capacity building and student nurture, under the leadership of Homi Bhabha Centre for Science Education (HBCSE). VP is focused on secondary level science & mathematics, primarily grades 8th to 10th, and has developed several Learning Units (LUs) which aim to provide rich pedagogic engagement to students and teachers. VPDS were online sessions for teachers, started during the pandemic and involved discussions on many LUs. Each VPDS session involved 2 virtual meetings where teachers participated in discussions about a particular LU and were expected to complete an assignment based on that LU. As the sessions were being conducted, we also hoped to reflect on the overall process and analyse some of the sessions for mathematics education research.

The LU ‘Exploring patterns in square numbers’ involved many opportunities for open mathematical explorations. Here we discuss teachers’ ideas of mathematical proofs while they engage in an open exploration through an analysis of their assignments. The assignment analysis is also useful to gain insights about how students can be supported during such open mathematical explorations.

ABOUT THE EXPLORATION TASKS:

This exploration has two different but connected tasks. In Task 1, a table of natural numbers and their squares is given and the teachers are asked to observe patterns in the table.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

Figure 1: *Snapshot of the table used in Task 1*

In Task 2, the table given below is shown where natural numbers up to 400 are arranged in a 8-column table (Figure 2) and the all the square numbers highlighted in the table. Teachers are expected to look for patterns.

I	II	III	IV	V	VI	VII	VIII		I	II	III	IV	V	VI	VII	VIII
1	2	3	4	5	6	7	8		209	210	211	212	213	214	215	216
9	10	11	12	13	14	15	16		217	218	219	220	221	222	223	224
17	18	19	20	21	22	23	24		225	226	227	228	229	230	231	232
25	26	27	28	29	30	31	32		233	234	235	236	237	238	239	240
33	34	35	36	37	38	39	40		241	242	243	244	245	246	247	248
41	42	43	44	45	46	47	48		249	250	251	252	253	254	255	256
49	50	51	52	53	54	55	56		257	258	259	260	261	262	263	264
57	58	59	60	61	62	63	64		265	266	267	268	269	270	271	272

Figure 2: *Snapshot of the table used in Task 2*

There was also a brief discussion on the processes of mathematics (Figure 3) and the distinction between simply verifying a conjecture for special cases as opposed to stating a concrete and general proof for the same.

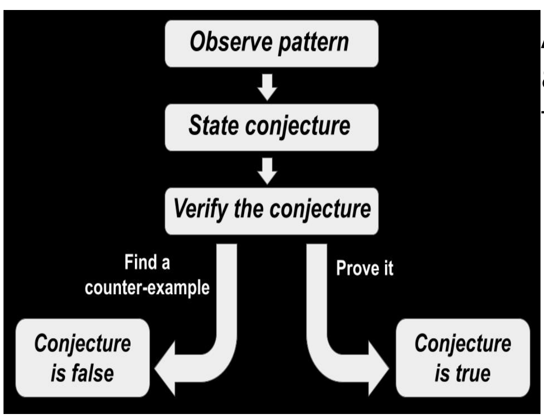

Figure 3: *Mathematical processes*

After the two sessions, teachers were given an assignment which consisted of the following two questions:

Q1. Look at this table (Figure 1) and find out two patterns. Verify them and prove or disprove them.
Q2. Look at this table (Figure 2) and find out two patterns. Verify them and prove or disprove them.

ABOUT THE ANALYSIS:

Here we will discuss some of the assignments submitted by teachers. We will analyse some of the proofs or non-proofs given by them and compare their answers to Q1. (based on Task 1) & Q2. (based on Task 2). The arrangement of numbers used in Task 1 was familiar to the teachers while the arrangement used in Task 2 was not. We categorised their responses into the following four categories:

1) Not attempted - if teachers submitted the assignment but did not respond to that particular question.

2) Observation - when teachers just report their observations/conjectures, which were neither verified or proved.

3) Verification - Here teachers verified their conjectures by substituting numbers, but did not prove their observed pattern.

4) Complete proof - when teachers correctly proved their conjectures, mostly algebraically.

Examples of assignments received from teachers are shown below. The one in figure 4a is categorised as 'observation' while the one in figure 4b is put under 'verification'.

Pattern 1:
Each row ends with multiples of 8

Pattern 2:
Row 2,3,5,6,7 dont have any squares

Pattern 3:
Only rows 1,4,8 have squares

Pattern 4:
Sum of rows is increasing arithmetically

Pattern 5:
As we move down the columns, we see less quantity of squares

We can show that:-
$(2n + 1)^2 - (2n - 1)^2 = 8n$

Put n = 1
$(2\times1 + 1)^2 - (2\times1 - 1)^2 = 8\times1$
$(3)^2 - 1^2 = 8$
$8 = 8$
LHS = RHS

Put n = 11
$(2\times11 + 1)^2 - (2\times11 - 1)^2 = 8\times11$
$23^2 - 21^2 = 88$
$529 - 441 = 88$
$88 = 88$
LHS = RHS

Figure 4a (L) and 4b (R): *Snapshot of teacher assignments.*

Our analysis indicates that in familiar scenarios (Task 1), teachers are likely to go beyond just observation and verification of patterns, and invest in presenting complete proofs. On the contrary, in unfamiliar situations, we observed a greater likelihood of teachers just reporting patterns.

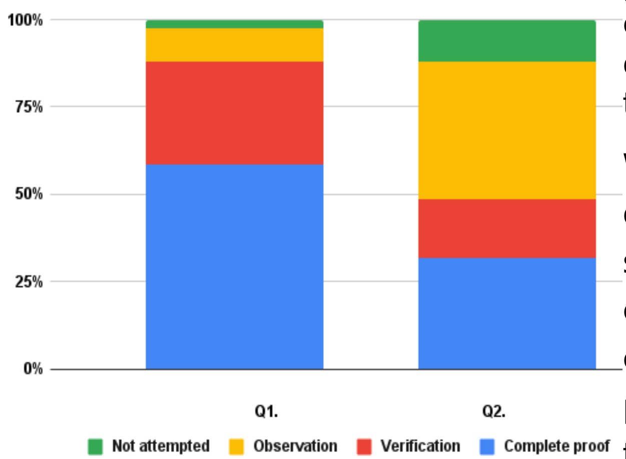

Figure 5: *Analysis of teachers' assignments*

What does the analysis inform in terms of pedagogic practices? While students' and teachers' participation can't be assumed the same, or compared directly, the analysis provides some idea about the response to open mathematical explorations. Based on this knowledge, we suggest, when such tasks are done with students, teachers may need to be proactive when discussing unfamiliar scenarios and be prepared to provide extra nudging to students to go beyond the observation stage.