Types of Proofs that Teachers Use While Engaging in an Open Mathematical Exploration

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All secondary mathematics teachers teach proofs in their classroom. But they rarely get an opportunity to think of new proofs, which are different from the ones they teach. To understand the types of proof teachers, use when they want to prove ‘new’ claims, we analysed assignments submitted by 41 secondary mathematics & science teachers post a virtual mathematics session on teacher capacity-building.

Keywords: Mathematics Education, Proofs, Open Exploration

Introduction

Proofs are central to the discipline of mathematics, but that cannot be said about the role of proofs in school mathematics. A lot of researchers have recommended that proofs should be central to school students’ mathematical experiences (Yackel & Hanna, 2003). School textbooks rarely give teachers & students opportunities to ‘prove’ (Wu, 1996). In secondary mathematics, proof or proving is often looked at as a formal activity isolated from the other mathematical activities done in the school (Stylianides, 2008).

About the Study and the Data

In order to improve the role of proofs in secondary school mathematics, activities which promote situations where teachers and students feel the need to prove and get opportunities to prove the ‘new’ or ‘unknown’ results need to be developed. The sets of tasks titled ‘Exploring Patterns in Square Numbers’ were conducted virtually with secondary mathematics & science teachers with the same objective.

The tasks

This exploration has two different but connected tasks.

Number	1	2	3	4	5	6	7	8	9	10
Square	1	4	9	16	25	36	49	64	81	100

I	II	III	IV	V	VI	VII	VIII		I	II	III	IV	V	VI	VII	VIII
1	2	3	4	5	6	7	8		209	210	211	212	213	214	215	216
9	10	11	12	13	14	15	16		217	218	219	220	221	222	223	224
17	18	19	20	21	22	23	24		225	226	227	228	229	230	231	232
25	26	27	28	29	30	31	32		233	234	235	236	237	238	239	240
33	34	35	36	37	38	39	40		241	242	243	244	245	246	247	248
41	42	43	44	45	46	47	48		249	250	251	252	253	254	255	256
49	50	51	52	53	54	55	56		257	258	259	260	261	262	263	264
57	58	59	60	61	62	63	64		265	266	267	268	269	270	271	272

Fig.1. Snapshots of the tables used in Task 1 (left) and Task 2 (right)

In Task 1, a table of natural numbers and their squares is given and the teachers are asked to observe patterns in the table. In Task 2, a table of natural numbers up to 400 are arranged in an 8-column table and some square numbers are highlighted (see Fig.1, right) & again the same set of teachers are asked to observe patterns in this table. The hour-long session discussed some of the patterns, after which the teachers were given an assignment comprising of the following three questions:

1. Look at this table (Fig.1 (Task 1)) and find out two patterns. Verify them and prove or disprove them.
2. Prove Pratibha's pattern
3. Look at this table (Fig.1 (Task 2)) and find out two patterns. Verify them and prove or disprove them.

For this study, we look at only Q1 and Q3. Q1 was based on Task 1 which used a familiar arrangement of numbers and their squares used regularly in textbooks, while Q3 was based on Task 2, which used an unfamiliar arrangement of numbers.

Theoretical Framework

Openness of the tasks: Yeo (2015) includes the following five elements: openness of answers, methods, complexities, goals and extensions for characterising open explorations. The tasks (Figure 1) are both open in answer and method. It is practically impossible to come up with an exhaustive list of all the patterns that individuals engaging in these explorations can think about. Patterns that emerge while doing these tasks vary in complexity making it open to complexities involved. Though the task has a very specific goal, namely "Finding Patterns", it does not specify what types of patterns and hence even in the parameter of goals, one can say that in this exploration there is an openness of goals. This arrangement of numbers that is used in Task 2 enables the facilitator to extend this task and find patterns when the numbers are arranged in different numbers of columns. So even in the parameter of extensions, this exploration fulfills Yeo's criteria of an open exploration.

Classification of the proofs: In their book, 'Proof in Mathematics Education' (Reid& Knipping, 2010), the authors proposed a framework for categorising proofs. They made four broad categories of proofs based on the use or non-use of different representations in them.
Empirical – specific examples are used but these examples do not represent a general class
Generic – specific examples are used as representations of a general class
Symbolic – words and symbols are used as representations
Formal – words and symbols are used but they do not represent anything
Using the above framework, we categorise different proofs given by teachers in the context of their familiarity with the arrangement of numbers in the task. In this study, we look at 41 secondary teachers' assignments from three different sessions. We choose two out of the three assignment questions to see the different kinds of proofs teachers think of and how these proofs depend on their familiarity with the arrangement of numbers.

Research Questions

The study aims to address the following research questions:

- What are the different types of proofs teachers give when they prove 'new' results?

- Does the choice of type of proof depend on the teachers' familiarity with the arrangement of numbers used in the tasks?

Method and Findings

We analysed some of the proofs given by the teachers participating in the online sessions using the framework proposed by Reid & Knipping (2010) and compared their proofs for Q1 (based on Task 1) with Q3 (based on Task 2).

	Symbolic proofs	Empirical proofs
Q1 (Task 1)	60.97%	39.02%
Q3 (Task 2)	34.15%	60.97%

Table 1. Distribution of symbolic & empirical proofs for Q1 & Q3

Conclusion

It is known that in different contexts students believe in the validity of the statement but are unable to express or analyse it (Balacheef, 1988). From our data, it seems that even in case of teachers, in contexts unfamiliar to them; they seem to believe in the validity of their patterns but are unable to express them symbolically. Hence the way a problem is represented impacts the type of proof teacher chooses to prove a 'new' result.

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Acknowledgments

The authors would like to thank the participating teachers for their support and cooperation. The authors also acknowledge the Vigyan Pratibha team for their support, help and suggestions in writing this analysis. We acknowledge the support of the Govt. of India, Department of Atomic Energy, under the Vigyan Pratibha Project (No. RTI - 4008).