Draw like da Vinci!

Introduction

Amber used to get many drawing tasks as a part of homework since primary school. For these tasks, he had to look at drawings in the textbook or from the internet, and copy the same image in his notebook. Some others found his drawings unappealing. So he used to ask his mother or father to make the drawings for him. He often wished that he found some tools which would help him in drawing some shapes or designs exactly as given in a book or on the internet.

One day, his elder sister, Rushda told him about these two amazing tools – a straight edge (any object with which one could draw a straight line like the edge of your geometric box or the ruler in your geometric box) and a compass. She said that these tools will help him draw the designs exactly. These two tools are also called Euclidean tools, named after the famous Greek mathematician, Euclid (325 BC - 265 BC).
So Amber also started making some designs using the two Euclidean Tools.

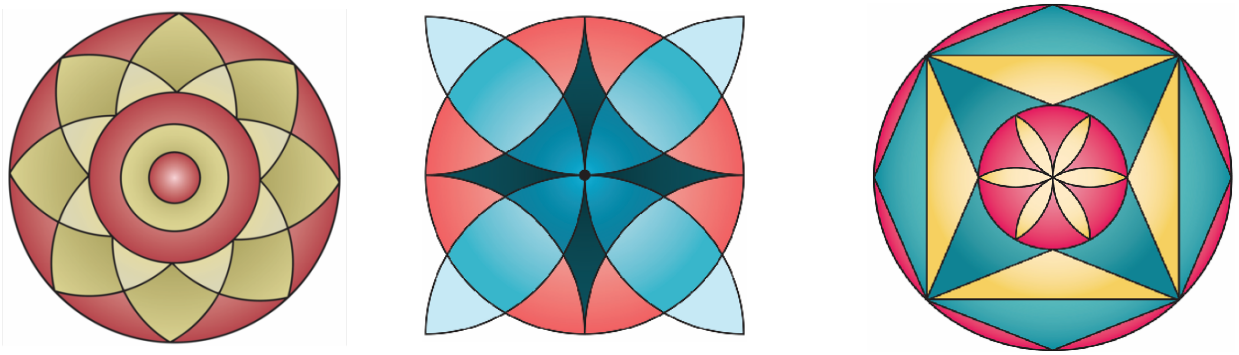


Figure 1: Amber's Designs

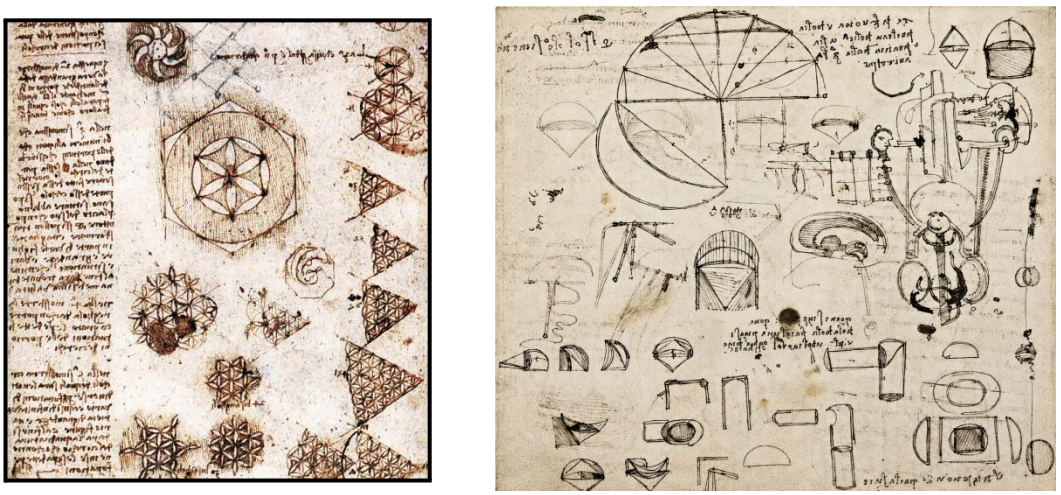


Figure 2: Geometric constructions tried by Leonardo da Vinci to make repeating patterns (Codex Atlanticus).

Looking at his drawings, his elder sister told him that his drawings were similar to the drawings made by the famous artist Leonardo da Vinci, who lived in Italy in the 15th century. She also showed him some other pictures made by Leonardo da Vinci which were very different from his but were made using the same two tools.

In this Unit we will try to make one of Amber's designs (Figure 3) using only Euclidean tools.

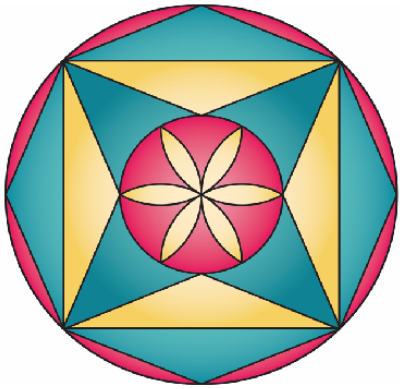


Figure 3

Look at Amber's design carefully.

1. What are the different shapes you see in Amber's design?

Task 1: Starting out

Let us start by drawing a square.

Q. 1.1. What is a square? What are different things you need to keep in mind while drawing a square?

We want to draw a square, for that we need to draw a line perpendicular to a given line segment without using a protractor.

Do not forget to label the vertices

i. Draw a segment AB.
ii. Draw a circle using a compass with centre at point A and radius more than half of length of segment AB.
iii. Now draw another circle of the same radius with its centre at point B.
iv. These two circles will intersect at two points. Label these points as C and D. Join points C and D using a line segment.

Q. 1.2. What is the angle between the segments AB and CD? Check using a protractor.

Q. 1.3. Can you justify the measure of the angle you obtained using geometric properties of different shapes? (Hint: Think which triangles you can draw/use to prove this)

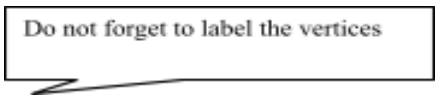
Q. 1.4. Where do the mid-points of the line segments, AB and CD lie? Check using a scale.

Q. 1.5. Will the answer to Q1.2, Q1.3 and Q.1.4 remain the same if one of the circles was of a smaller radius than the other?

Q. 1.6. Now given a line segment PQ, can you draw the **perpendicular bisector of PQ**?

Task 2: Constructing a square and a circle passing through its vertices

Q. 2.1. Can you use the technique of drawing a perpendicular line to a given line segment, to construct a square? How will you do it?



i. Start with a segment AB.
ii. Draw perpendicular lines to the line segment passing through points A and B.
iii. Draw a square ABCD.

Q. 2.2. Prove that ABCD that you have constructed in a square.

Now we have to draw a circle such that the circle passes through all the vertices of the square ABCD.

Q. 2.3. How will you draw such a circle? How will you determine the center and the radius of this circle? Will this circle touch all the four vertices of the square? Why?

Task 3: Constructing the angles at the vertices of the square:

Q. 3.1. What is the measure of each of the angles in Amber's design? Justify your answers and then use a protractor to verify your answers.

Now, you have to construct these angles without a protractor.

Q. 3.2. Can you think of a measure of an angle which can be constructed without a protractor?

Q. 3.3. Are there any other angles you can draw without a protractor?

To be able to answer these questions better, let us learn another construction by which we can bisect a given angle.

i. Draw an angle of any measure.
ii. Mark the vertex of the angle, call it Q. With Q as the centre, draw a circle intersecting the two arms of the angle.
iii. Call these two intersection points P and R.
iv. Take the same radius and draw two more circles from P and R which will intersect at a point. Call this point S.
v. Draw a line joining the intersection points of the circles, S and the vertex of the angle, Q.

Q. 3.4. What do you think this line is? How can you check if your guess is correct?

Q. 3.5. Why is the line obtained through the construction, the bisector of the given angle?

You must have noticed that drawing whole circles makes the construction look very confusing and crowded. In a lot of constructions, we need not draw the whole circle but draw only small parts of a circle, called arcs, to get intersection of the circle with other objects. Sometimes drawing only arcs is enough.

Q. 3.6. Is it possible to make the angles given below using only a straight edge and compass (the Euclidean tools), and without using a protractor?

(a) 60°

(b) 120°

(c) 90°

(d) 45°

(e) 67.5°

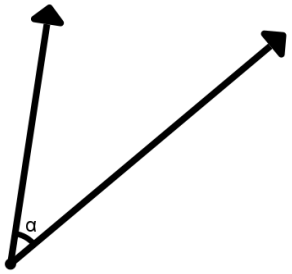
Q. 3.7. What were the methods that you used?

Q. 3.8. How can you be sure that the angle you drew is of the given measure?

Q. 3.9. List out three more angles, other than the ones given above, which you can make using only a straight edge and a compass. How will you make them? Justify why the measures are correct.

So, until now we found out how to make angles of some measures and also bisect a given angle. Now we need to figure out how to copy these angles.

Q. 3.10. Draw a given angle, α, of any measure. Now, using arcs, draw another angle that has the same measure as the given angle, α. How did you do it? Share your method with your friends.



Q. 3.11. Prove that these two angles will be equal.

Q. 3.12. Will your construction for copying any angle work even for copying obtuse angles?

Task 4: Making an octagon

We want to make an octagon.

Q. 4.1. Is the octagon in Amber's design a special kind of octagon? What is the measure of each angle of this special type of octagon? Justify your answers.

Q. 4.2. What is the difference between the measure of one angle of this octagon and a square?

Q. 4.3. If we have a square, can we make this special octagon using the four vertices of the square? How?

Do not forget to label the vertices

Task 5: Making a 6-petalled flower in a circle.

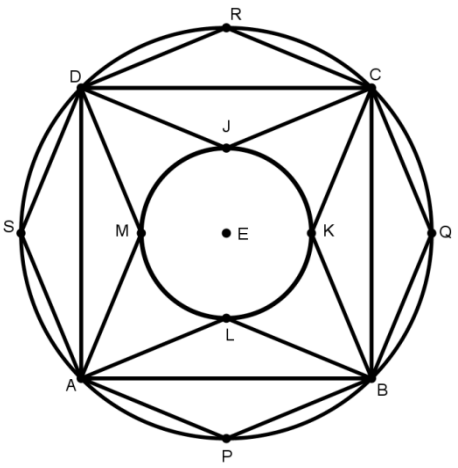
Draw a circle using a compass with the centre at O. We want to draw a 6 petalled flower inside this circle.

i. Draw another circle of the same radius with its centre O_1 anywhere on the **first circle's boundary**.
ii. The two circles drawn will intersect at two points. Label one of these points as O_2.
iii. Draw another circle of the same radius with its centre at O_2. This circle will intersect the circle at O at two points, O_1 and another new point. Label the new point, O_3.
iv. Draw four more circles. Now you must have six circles intersecting the circle at O.
v. Erase the parts of the circles outside the circle at O.

Q. 5.1. Did you get a 6-petalled flower?

Do not forget to label the vertices

Task 6: Making Amber's design



i. Draw a fresh square following the procedure in Task 3. Label it ABCD.
ii. Draw the two diagonals AC and BD. Let them intersect at E.
iii. Next draw a circle with a centre at the intersection of the two diagonals and passing through one of the vertices of the square.
iv. You get eight angles (like CAB) of measure 45° each.
v. Draw angle bisectors of all these eight angles such that the two bisectors of adjacent angles intersect at points J, K, L, and M, respectively.
vi. On the outer sides of the square draw lines from points A and B each making an angle of 22.5° with segment AB and intersecting at point P.

(Hint: Note that one of the angles inside the square has a measure of 22.5°, which you can use for this construction)

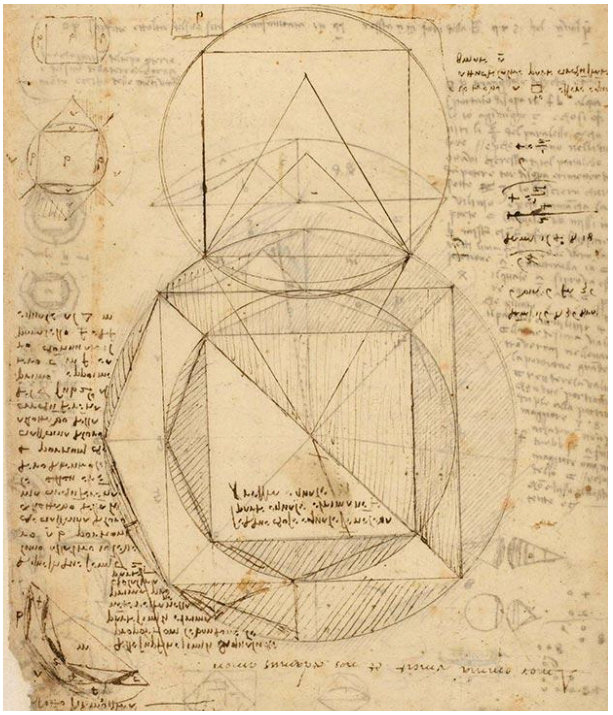
vii. Similarly, draw 6 more lines making angles of 22.5° with the other three sides of the square intersecting at points Q, R and S. These eight lines will define the octagon, APBQCRDS.
viii. Erase all the lines and arcs which are not necessary.
ix. Draw a circle with centre, E passing through one of the four points, J, K, L, and M.

x. Draw the 6-petalled flower in this circle.

Q. 6.1. Did you get Amber's design? Is it exactly the same design?

You can do some shading in the design you got or draw more intricate designs in it.

Q. 6.2. Instead of starting from square, is it possible to first draw the outermost circle and then draw other components of the Amber's design without drawing the square in the initial steps?



A geometrical construction by Leonardo da Vinci (Codex Atlanticus).